ON THE PREDICTION OF MAXIMUM AMPLITUDE FOR SOLAR CYCLES USING GEOMAGNETIC PRECURSORS

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Abstract. Precursor methods for the prediction of maximum amplitude of the solar cycle have previously been found to provide the most reliable indication for the size of the following cycle, years in advance. In this paper, we evaluate several of the previously used geomagnetic precursor methods and some new ones, both as single-variate and multivariate regressions. The newer precursor methods are based on the size of the geomagnetic index maximum, which, since cycle 12, has always occurred during the declining portion of the solar cycle, usually several years before subsequent cycle minimum. These various precursor techniques are then applied to cycle 23, yielding the prediction that its maximum amplitude should be about 168 ± 15 (r.m.s.), peaking sometime in 1999−2000.

1. Introduction

In 1966, Ohl noticed a high correlation between the geomagnetic activity cycle minimum and the maximum amplitude of the following sunspot cycle (Ohl, 1966). This behaviour suggests the existence of a 'precursor' relationship active in sunspot cycles. Furthermore, a few years later, he showed that the level of geomagnetic activity during the last years of a sunspot cycle also is well correlated against the amplitude of the following cycle (Ohl, 1968, 1971, 1976; Ohl and Ohl, 1986). During the last years of a cycle, the geomagnetic activity results from recurrent storms, fast solar winds, and coronal holes (Ohl, 1971; Svalgaard, 1977; Legrand and Simon, 1981). Ohl's results have been interpreted as suggesting the notion of the 'extended cycle' (e.g., Wilson, 1994), a concept now widely accepted, in which the sunspot cycle actually begins several years before new cycle minimum, near maximum of the old cycle.

In addition to methods to extrapolate the evolution of the current cycle (e.g., McNish and Lincoln, 1949; Waldmeier, 1968; Wilson, 1990b; Lantos, 1990; Macpherson, 1993; Fessant, Pierret, and Lantos, 1996), many authors have proposed methods for the prediction of the size of the next sunspot cycle (see the comprehensive analysis by Denkmayr (1993) and summaries for cycle 23 by Obridko (1995) and Kane (1997). Comparisons (Brown and Simon, 1986; Kunches, 1993) for the last two cycles (21 and 22) show that those methods based on observed precursors

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perform the best globally, and they are the methods retained for operational purposes (Jocelyn *et al.*, 1997). In addition to geomagnetic activity, a number of direct precursors have been tested (Schatten *et al.*, 1978; Layden *et al.*, 1991; Schatten and Pesnell, 1993; Schatten, Myers, and Sofia, 1996; Bravo and Stewart, 1997), but, possibly due to difficulties of precise measurements, their correlation with $RI_{\max}$ is lower than with geomagnetic precursors (Layden *et al.*, 1991). In addition to linear regression analysis, neural networks have been used to predict the next solar cycle (Calvo, Ceccato, and Piacentini, 1995; Tian, 1996).

The purpose of this study is to compare various methods for the prediction of maximum amplitude of the solar cycle, especially those based on geomagnetic activity indices. This last group of methods, despite wide differences of epoch taken into account, gives highly coherent results. We are using these techniques to estimate the likely size of cycle 23. Our analysis indicates that cycle 23 will have a maximum amplitude (in terms of smoothed monthly mean sunspot number) that will be larger than average, very likely, comparable to or larger than that seen for cycle 22. Specifically, our analysis yields the prediction that cycle 23 should have a maximum amplitude measuring about 168 ± 15 (r.m.s.) and that it should peak sometime in 1999−2000.

2. Methods for Predicting Maximum Amplitude

The current methods of geomagnetic precursor prediction differ from that originally proposed by Ohl, although they closely follow from his results. Many associate the size of the cycle in terms of maximum sunspot number with geomagnetic precursors observed during the minimum of the cycle. They include minimums of geomagnetic data (Gonzalez and Schatten, 1987; Kane, 1987, 1989; Wilson, 1990a; Layden *et al.*, 1991), number of Anomalous Quiet Days (AQD, see Brown and Williams, 1969; Brown, 1979) and number of Disturbed Days (NDD, see Wilson, 1990a). During the minimum of the cycle, bivariate analysis with minimums of geomagnetic indices and minimums of sunspot numbers have been proposed by Sargent (1978), Wilson (1988a,b, 1990a), and Kane (1989).

During the declining phase of the cycle, authors have proposed techniques to separate geomagnetic activity related to sunspot activity from the recurrent geomagnetic activity. Some prediction methods use estimates of the geomagnetic activity obtained by identification of individual recurrent storms (Legrand and Simon, 1981; Thompson, 1985), other methods subtract a component proportional to sunspot numbers (Ohl, 1968; Kataja, 1986; Li, 1997). Indeed, Feynman (1982) showed that the geomagnetic activity could be separated into two components, one being proportional to sunspot number and the other being the residual. Kataja (1986) showed that the residuals correlate with the size of the following cycle amplitude. By counting the number of disturbed days (NDD) with $Ap \geq 25$ over a cycle, N_c, Thompson (1993) found that it is possible to predict the maximum of the next sunspot cycle RI_n after subtraction of the last sunspot maximum RI_c. Indeed Thompson showed a correlation between N_c and $RI_c + RI_n$ (same weight for RI_c and RI_n). Bivariate analysis of N_c versus RI_c and RI_n separately gives the same result at the precision available.

When only the last years of the solar cycle are considered, separation of geomagnetic activity due to active regions from recurrent storm activity is not necessary because recurrent activity is dominant during this period (Legrand and Simon, 1981; Hedeman and Dodson-Prince, 1986). Average values of the geomagnetic indices over the last years of the cycle have been used as precursors of the following cycle maximum by Ohl (1976), Wilson (1990a), Denkmayr and Cugnon (1996), Bounar, Cliver, and Boriakoff (1997), and Jain (1997).

A rather simple method involving the late maximum of geomagnetic activity (noted aa^*_{max} for the aa indices) is proposed here. We have chosen aa indices, rather than Ap indices because of the longer observations of the former. As the late maximum occurs during the last years of the cycle, we are not subtracting sunspot numbers. That is the main difference with the methods used by Kataja (1986) or Li (1997). Li (1997) has used the Ap index with a linear regression over five cycles to predict cycle 23. Unlike us he introduced a supplementary 35-month running average to smooth the late maximum of the Ap index. Ohl (1971) recognized that the size of the following cycle was statistically associated with the size of the recurrent maximum. Simon (1979) has also proposed using the late maximum of geomagnetic activity, but his method assumes that this maximum will be at the same level as the geomagnetic activity during sunspot maximum (which is only true as a first approximation). In Figure 1 we compare temporal profiles of geomagnetic activity (in terms of smoothed values) to sunspot numbers, for cycles 11 to 22. The late maximum of geomagnetic activity for each sunspot cycle (indicated by the asterisk) is found to correlate with the maximum amplitude for the next cycle; i.e., it was lower in November 1963, before cycle 20 ($RI_{max} = 110.6$) than in September 1974, before cycle 21 ($RI_{max} = 164.5$) or in September 1984 before cycle 22 ($RI_{max} = 158.5$). Figure 1 shows that the maximum value aa^*_{max} for cycle 23 is similar to the amplitude of both cycles 21 and 22. Note nevertheless that over 12 cycles, the late maximum is missing during one cycle: Figure 1 shows that no maximum is detected during the five last years of cycle 11. Table I gives dates and amplitudes of aa^*_{max} for cycles 12 to 22, as well as dates of minimums of RI_{12} and aa_{12}, for comparison.

Figure 2 displays the scatter plot and regression coefficient, r, for the aa index of the late maximum amplitude, aa^*_{max}, versus the maximum sunspot number of the next cycle RI_{max} (on the left side of the figure). For comparison, the figure displays the same diagram for the minimum aa index, aa_{min} (on the right side of the figure), which is one of the frequently used geomagnetic precursors. Figure 2 shows that the aa^*_{max} geomagnetic precursor, in terms of linear regression, performs better than aa_{min}, with a regression coefficient r equal to 0.961, instead of 0.891 for aa_{min}, when cycles 13 to 22 are taken into account.

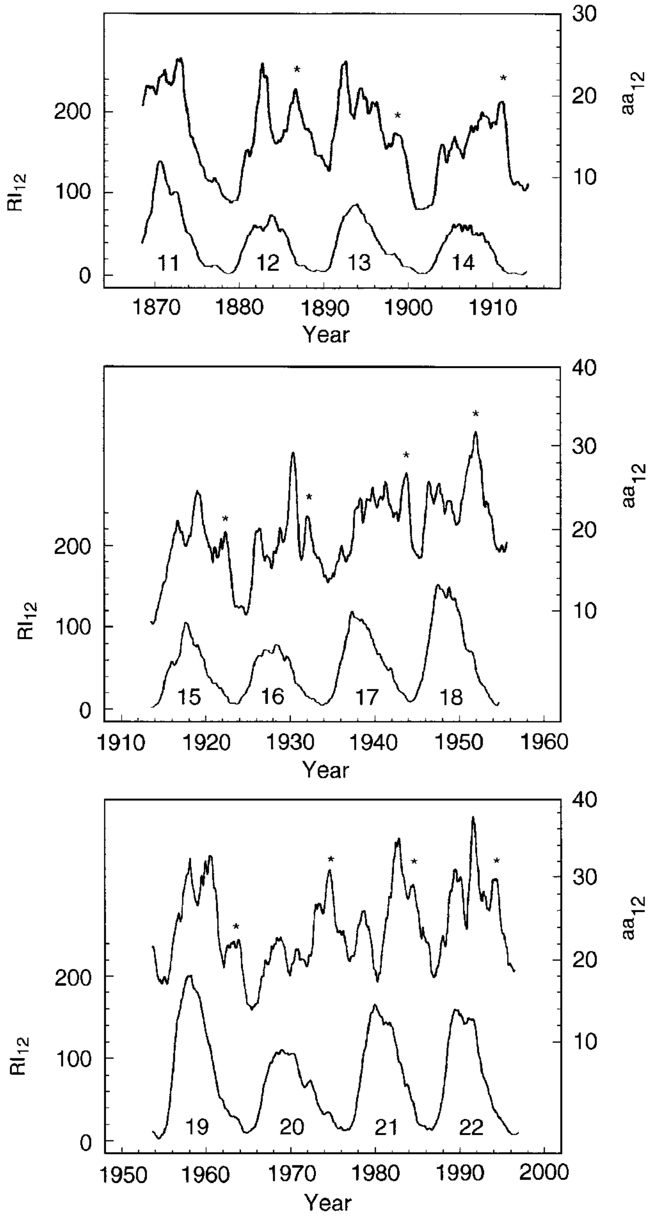

Figure 1. Comparison of the geomagnetic *aa* cycles (upper curves) with the sunspot number cycles (lower curves) from 1868 to 1996. Late maximum precursors aa^*_{max} are indicated with asterisks.

TABLE I

Sizes and dates of late *aa* maximums and dates of *RI* and *aa* minimums

Cycle	aa_{max}	aa^*_{max} date	RI_{min} date	aa_{min} date
12	20.90	Aug. 1886	Mar. 1890	July 1890
13	15.24	Dec. 1898	Jan. 1902	Dec. 1900
14	19.40	Jan. 1911	July 1913	Sep. 1913
15	19.60	May 1922	Aug. 1923	Oct. 1924
16	21.53	Dec. 1931	Sep. 1933	June 1934
17	26.70	Oct. 1943	Feb. 1944	Apr. 1945
18	31.82	Dec. 1951	Apr. 1954	Oct. 1954
19	22.51	Nov. 1963	Oct. 1964	May 1965
20	30.90	Sep. 1974	Mar. 1976	Dec. 1976
21	29.07	July 1984	Sep. 1986	Dec. 1986
22	29.80	May 1994	May 1996	June 1997

3. Evaluation of the Methods

The comparison of only one cycle prediction, as Brown and Simon (1986) and Kunches (1993) have done for cycles 21 and 22, is not sufficient to evaluate the individual methods because of the uncertainty of a single prediction, including the dependence upon the cycles taken into account to compute the linear regression. Thus, the predictions for the sample of past cycles have to be more systematically tested.

To compare the level of skill of different methods in predicting maximum sunspot number, it is also necessary to apply the methods on data as homogeneous as possible. Thus we are using for RI_{max} and for all geomagnetic precursors smoothed monthly values A_{12} obtained from the monthly averages A_m according to the following (Waldmeier, 1961):

$$A_{12} = \frac{1}{12}\left(\sum_{m=-5}^{m=+5} A_m + \frac{A_{-6}}{2} + \frac{A_{+6}}{2}\right) .$$

We use the smoothed monthly values rather than annual values, frequently used in the past literature, because the latter suffers from a selection effect related to the calendar and, in our opinion, its use is justified only if monthly data are not available.

The geomagnetic indices used here are on the one hand the *aa* antipodal indices, available since 1868 (Mayaud, 1980), and computed for the recent decades by the

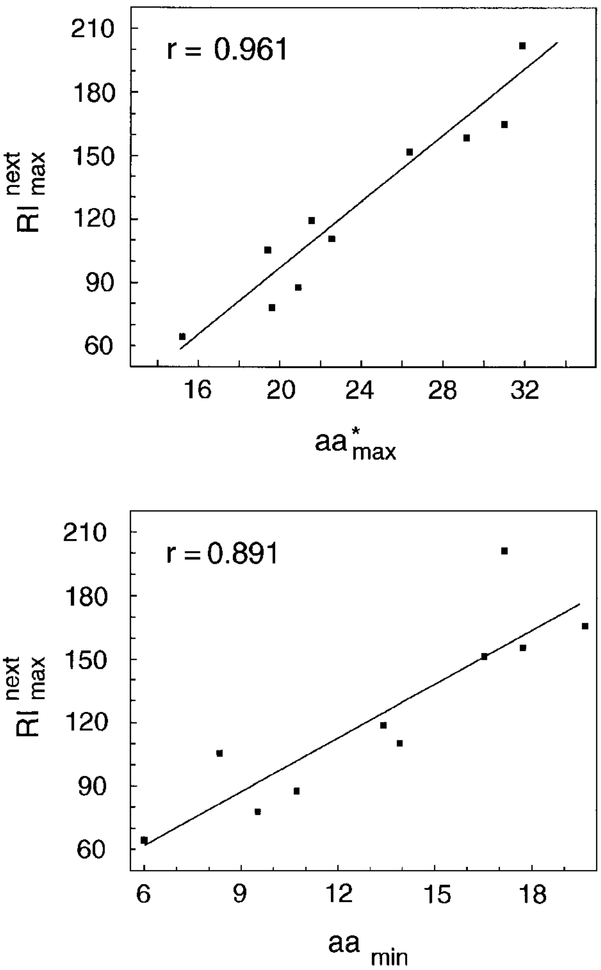

Figure 2. Linear regression for prediction of RI_{max} of the next cycle with aa^{*}_{max} (*above*) and with aa_{min} (*below*).

ISES Paris Warning Center on behalf of the International Service of Geomagnetic Indices, and on the other hand, the Ap planetary indices (Bartels, 1949) computed by Institut für Geophysik Göttingen and Geoforschungszentrum Potsdam.

In this section we consider predictions of the same cycles, from 13 to 22, for all the methods. Note that regression coefficients could be compared only if the number of points is the same. As Ap geomagnetic indices are available since 1932 only, an extrapolation is done to the past, using a non-linear regression from the Ap versus aa index, as suggested by Mayaud (1980). The regression law found here on smoothed monthly values and with the data from 1932 to 1995, is

$$\hat{A}p = 0.00222aa^2 + 0.6814aa - 2.27 ,$$

with a regression coefficient of 0.9625, a mean difference between estimation and actual index of 0.36 and a standard error of 1.59.

For some of the methods mentioned in the previous section, the data updated to cycle 22 are not available. This is the case for the methods proposed by Brown and Williams (1969), Legrand and Simon (1981), and Thompson (1985). The method involving the number of disturbed days during the minimum of the cycle (Wilson, 1990a) corresponds, according to the author, to a significantly lower regression coefficient than for the other methods he tested.

Thus we are comparing here the precursor aa^*_{max}, the late maximum of geomagnetic activity, to six other precursors, namely:
aa_{min} and Ap_{min}, minimum values of the geomagnetic indices;
aa_{36} and Ap_{36}, average values of the indices over the last three years of the cycle;
aa_F corresponding to the aa index after subtraction of the sunspot-cycle-related component studied by Feynman (1982),

$$aa_F = aa - 0.12RI_{12} - 5.4 ,$$

to predict the maximum of the next cycle RI_{max}; and finally NDD, the number of disturbed days ($Ap \geq 25$) over the entire cycle (Thompson, 1993) to predict $RI_c + RI_n$, the sum of the current and of the next cycle maximum sunspot numbers. Here the counts of days with $Ap \geq 25$ are extended to the end of cycle 22.

Figure 3 shows, on the left side, the histograms of the regression coefficients obtained with single-variate analysis for the prediction of the past cycles (i.e., removing the precursor in turn for each cycle to be predicted). With RI_{min} as a second independent variable, the bivariate analysis gives the histograms shown on the right side, from the best average regression coefficient at the top to the lowest at the bottom. In most cases, the bivariate analysis significantly improves the prediction method. The best method at this level is found to be with bivariate analysis with aa^*_{max} and RI_{min} as precursors, with an average regression coefficient of 0.970. Improvement of predictions with bivariate analysis has been already shown by Wilson (1988a,b, 1990a) and Kane (1989) for the methods involving aa_{min} and Ap_{min} as precursors.

Figure 4 shows similar histograms obtained when three variables, different in nature or involving different epochs of the cycle, are used. On the top the variables are aa^*_{max}, aa_{min}, and RI_{min} (the method hereafter called Mult 1) and at the bottom they are aa^*_{max}, Ap_{min}, and RI_{min} (hereafter called Mult 2). In both cases, regression coefficients greater than 0.975 are obtained, showing that the methods with three variables perform the best with respect to this criterion. For the methods coupling three precursors, the histograms of the difference between prediction of RI_{max} and observation is given for each cycle from 13 to 22 on the right side of Figure 4.

Indeed the distribution of regression coefficients given in Figures 3 and 4 provides a rather general criterion to estimate the skill of the methods: its shows how robust the method is. Nevertheless this criterion alone is not sufficient: it is also

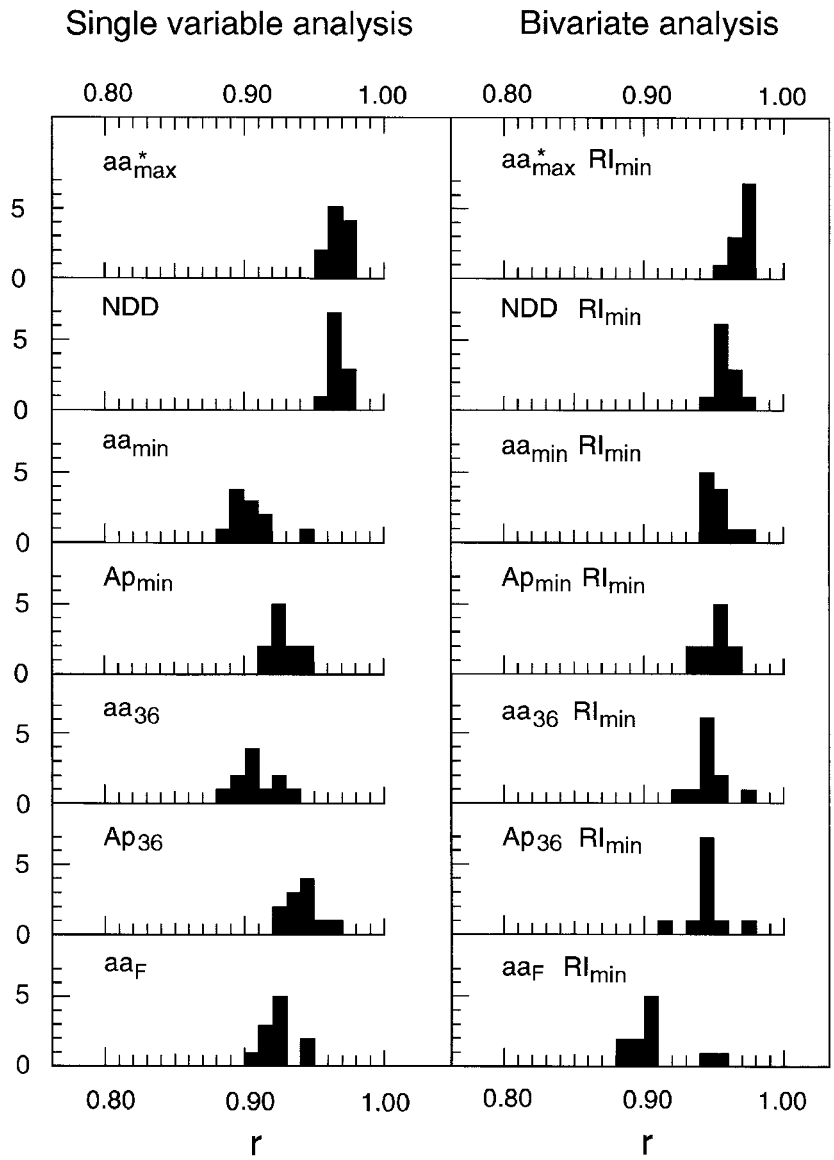

Figure 3. Histograms of the regression coefficients *r* for prediction of cycles 13 to 22. For each cycle to be predicted, the precursor has been removed from the regression analysis, so the regression coefficient differs. On the left with single-variate analysis, on the right with bivariate analysis. The precursors are indicated in each box. The different methods are given with decreasing order of the average regression coefficient with bivariate analysis.

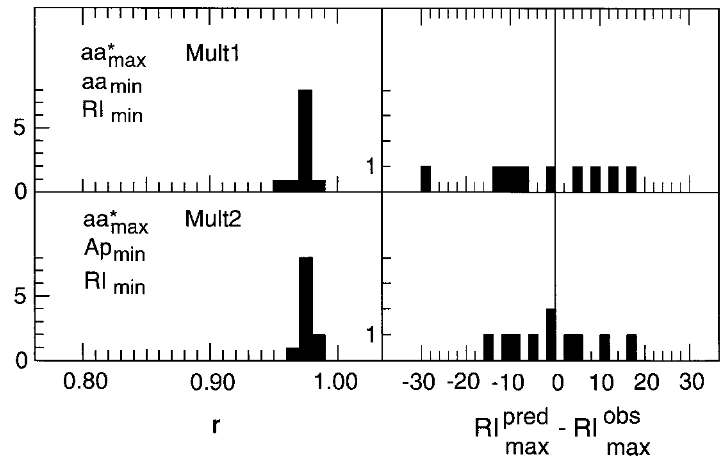

Figure 4. Histograms of the regression coefficients r for prediction of cycles 13 to 22 with three independent variables (*left*). Histograms of the differences between predictions and observations (*right*).

useful to compare the errors made when the different methods are applied to the 'prediction' of past cycles, despite the limited number of available cycles.

Table II summarises, for all the methods studied here, the average errors of the prediction, as well as maximum errors, in addition to the average regression coefficients, all for cycles from 13 to 22. As pointed out above, the method with NDD as precursor has, compared with other methods, a different dependent variable ($RI_c + RI_n$) instead of $RI_{\max}$ of the next cycle. Its regression coefficient is among the highest, but it is one of the less efficient in terms of error of the prediction.

If we now consider the other methods, all with $RI_{\max}$ as dependent variable, we see from Table II that the situation is different for single-variate analysis and for multivariate analysis. In the first case, the rank of the methods is almost similar with both criteria, either the average regression coefficient or the average prediction error. With single-variate analysis, we could see from Table II that the method involving $aa^*_{\max}$ performs the best, followed by methods using Ap indices and finally those using aa indices.

With bivariate analysis, the ranks according to both criteria differ, but, as shown on Figure 3, the regression coefficients remain quite close for the different methods. With bivariate analysis, the methods could be separated into three groups: $aa^*_{\max}$ appears to be the best precursor, then four methods with $aa_{\min}$, $Ap_{\min}$, aa_{36}, and Ap_{36} give very close results and finally aa_{F} is less efficient.

Multiple regression with three variables improves the results slightly when Ap is used in addition to $aa^*_{\max}$ and $RI_{\min}$. Note that in this case, as well as with bivariate analysis with $aa^*_{\max}$ and $RI_{\min}$, the maximum error on the predicted $RI_{\max}$ for cycle 13 to 22 is lower than 20.

TABLE II

Skill of prediction methods: single-variate analysis.

Precursor	Average regression coefficient	Rank	Average error	Rank	Maximum error	Rank
aa^*_{max}	0.957	2	13.34	1	24.46	1
aa_{min}	0.906	7	17.60	6	53.88	7
Ap_{min}	0.930	4	16.18	3	43.10	3
aa_{36}	0.910	6	16.95	4	46.87	5
Ap_{36}	0.942	3	13.91	2	32.79	2
aa_F	0.924	5	17.16	5	44.67	4
NDD	0.967	1	18.06	7	48.54	6

Bivariate analysis

Precursor in addition to RI_{min}	Average regression coefficient	Rank	Average error	Rank error	Maximum	Rank
aa^*_{max}	0.970	1	9.29	1	17.16	1
aa_{min}	0.954	3	15.29	4	35.14	5
Ap_{min}	0.950	4	17.23	5	31.48	3
aa_{36}	0.946	5	14.37	3	27.43	2
Ap_{36}	0.945	6	12.24	2	32.15	4
aa_{36}	0.907	7	20.53	6	57.60	7
NDD	0.959	2	21.66	7	55.32	6

Three-variable analysis

Precursor in addition to aa^*_{max} and RI_{min}	Average regression coefficient	Rank	Average error	Rank error	Maximum	Rank
aa_{min}	0.976	1	11.23	2	28.22	2
Ap_{min}	0.975	2	8.01	1	16.52	1

TABLE III

Summary of single-variate predictions for the maximum amplitude of cycle 23.

Precursor	Intercept	Slope	Regression coefficient	Stand. error	Precursor cycle 22	Prediction cycle 23
NDD	36.93	0.460	0.967	19.18[a]	612	159.7
aa^*_{max}	−55.51	7.559	0.961	12.84	29.80	169.8
Ap_{36}	30.00	8.100	0.942	15.69	14.34	146.2
Ap_{min}	37.35	12.124	0.928	17.28	8.53	140.8
aa_F	−4.15	8.073	0.922	17.95	20.70	163.0
aa_{36}	9.85	6.012	0.908	19.42	24.42	156.7
aa_{min}	12.85	8.409	0.903	19.90	16.71	153.4

[a] The dependent variable is the sum of the current and next sunspot maximum indices.

From the comparison of methods involving the late maximum of geomagnetic indices, namely the method proposed here with aa^*_{max} as precursor, and the method (aa_F) derived by Kataja (1986) from the Feynman (1982) results, it appears unambiguously that the subtraction of a component proportional to sunspot number does not improve the predictions. For operational purposes, in addition to the skill of the methods, the regular availability of the precursors as well as the precedence of the prediction are also to be taken into account. Minimums of geomagnetic indices are generally delayed about half a year, compared to RI_{12} minimum of a cycle. Thus methods with aa_{min} and Ap_{min} are available later than when aa_{36}, Ap_{36},or NDD are used. On the other hand, methods with late geomagnetic activity maximum (aa^*_{max} and aa_F) give the prediction two years, on average, before the end of each cycle (see Table I), provided they are applied only with single-variate analysis.

4. Results for the Prediction of Maximum Amplitude for Cycle 23

According to its conventional definition, cycle 23 began in May 1996 and its maximum will be in 1999, as predicted with the duration of ascending phases of the similar cycles 21 (42 months) and 22 (34 months), or in 2000 according to many authors. The minimum of geomagnetic activity was observed in November 1996 for aa_{12} and in October 1996 for Ap_{12}. In the case of single-variate analysis, Table III summarises for each of the methods (with cycles 12 to 22, except for aa^*_{max}) the intercept, the slope of the regression line, the correlation coefficient, and the standard error, as well as the value of the specific indicator from cycle 22 and the resulting prediction of the maximum sunspot index RI_{12} for cycle 23. Methods are given according to decreasing order of the regression coefficient. Table IV does the same in the case of bivariate regression analysis.

TABLE IV

Summary of bivariate predictions for the maximum amplitude of cycle 23

Precursor in add. to R_{min}	Constant	A (precursor)	B (RI_{min})	Regression coefficient	Stand. error	Precursor cycle 22	Prediction cycle 23
aa^*_{max}	−65.92	8.756	−2.845	0.978	9.13	29.80	171.4
NDD	30.67	0.444	1.032	0.960	20.00 [a]	612	159.4
aa_{min}	1.08	12.018	−5.721	0.954	13.24	16.71	154.4
Ap_{min}	39.08	15.229	−3.830	0.950	13.73	8.53	137.2
aa_{36}	−0.587	8.242	−5.090	0.948	14.05	24.42	158.4
Ap_{36}	31.33	9.358	−2.559	0.947	14.08	14.34	144.3
aa_F	−4.79	8.437	−0.828	0.904	18.80	20.70	163.0

[a] The dependent variable is the sum of the current and next sunspot maximum indices.

When three variables are used in multivariate analysis, the RI_{max} is obtained with

$$RI_{max} = -53.20 + 3.015aa_{min} + 6.765aa^*_{max} - 3.710RI_{min} \, , \qquad \text{(Mult 1)}$$

with a regression coefficient $r = 0.978$ and standard error of estimate s.e. $= 9.11$, and

$$RI_{max} = -43.86 + 3.899Ap_{min} + 6.754aa^*_{max} - 3.260RI_{min} \, , \qquad \text{(Mult 2)}$$

with a regression coefficient $r = 0.978$ and standard error of estimate s.e. $= 8.99$.

The predictions obtained with both formulas are: $RI_{max}(23) = 168.0$ with aa_{min} as precursor and $RI_{max}(23) = 163.6$ with Ap_{min} as precursor.

Comparison of Tables III and IV shows that, for each precursor, predictions with single-variate and with bivariate analysis are extremely close to each other (this is not a general rule for other cycles). Thus it is sufficient, as in Figure 5, to compare the various predictions obtained with multivariate (2 and 3-variable) analysis. Figure 5 shows that the predictions obtained with the methods involving geomagnetic precursors are very close to each other: all are within a range from 140.8 to 169.8 (i.e., ±12% around their average value). As results are shown in decreasing order of the correlation coefficient, from left to right, we may consider as the best guess those methods located at the left of the figure. The result of the first three methods suggests a predicted value of RI_{12} for cycle 23 of 168 ± 15 (r.m.s.).

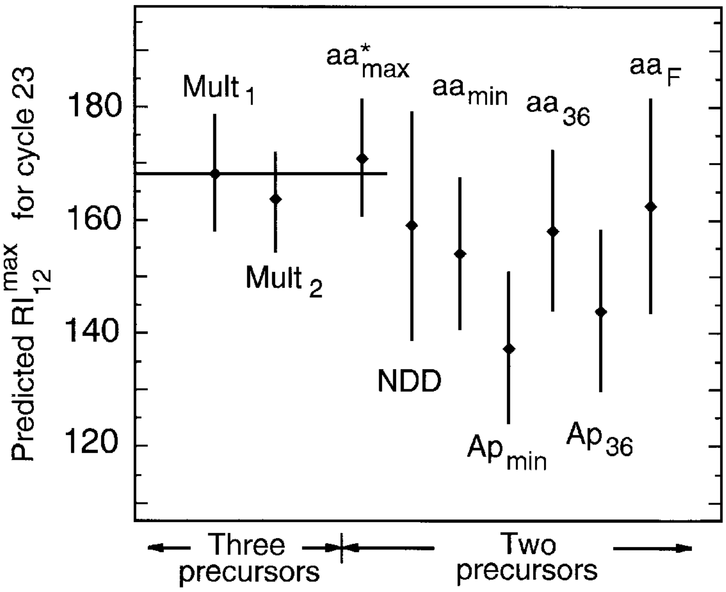

Figure 5. Predictions of the maximum of cycle 23. On the left, predictions with three independent variables (Mult 1 with aa^*_{max}, aa_{min}, and RI_{min}; Mult 2 with aa^*_{max}, Ap_{min}, and RI_{min}). On the right with bivariate analysis (the given precursor and RI_{min}). The bivariate analysis methods are given with decreasing order of the regression coefficient. The error bars are standard errors of estimate. A horizontal line gives the prediction obtained with the three best methods.

5. Discussion and Conclusion

Wilson (1990a) tested three methods studied here: those with smoothed monthly values of aa_{min}, Ap_{min}, and Ap_{36} as precursors, with single-variate and bivariate analysis. Kane (1989) compared methods with aa_{min} for both single-variate and bivariate analysis. They founded, in agreement with our results for other methods, improvement of the regression coefficient when bivariate analysis is used.

Denkmayr and Cugnon (1996), Kane (1997), and Li (1997) have given predictions of maximum of cycle 23 with different methods involving geomagnetic precursors. The first two used only provisional precursors, in advance of the end of cycle 22. Nevertheless the predictions are in agreement with our results, as shown in Figure 6. For example, Kane (1997) uses an aa_{min} provisional value of 17.9, instead of 16.7 now observed, and thus his prediction with single variate analysis would be now 159.5 instead of 170. In this figure the predictions based on geomagnetic precursors are given with bins in black. The bins numbered from 1 to 9 are the results obtained here, as detailed in the figure caption. All published predictions based on geomagnetic precursors (except a very low prediction by Obridko, 1996) are within the range 137–177, which represents only ±13% around their average value, which is equal to 157. Predictions for RI_{max} of cycle 23 with other precursors are indicated in Figure 6 with hatched bins. They are very close to the

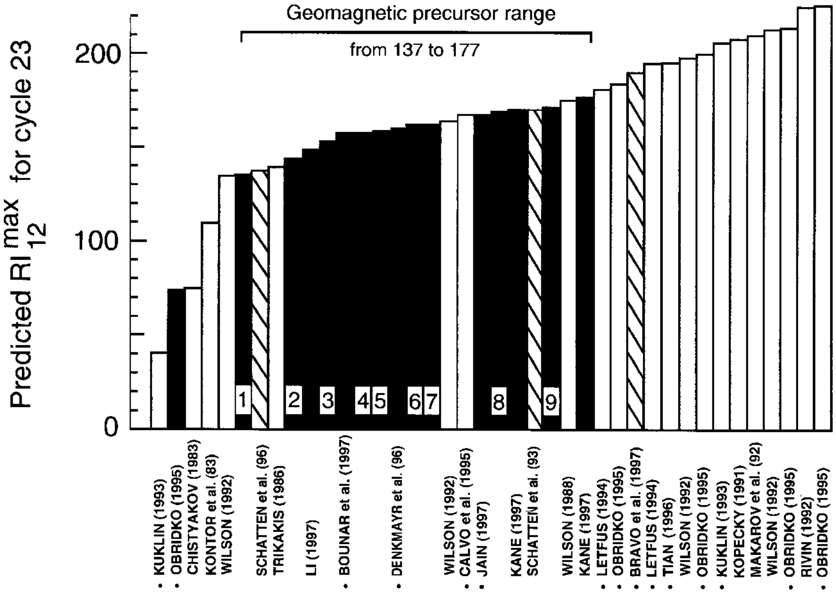

Figure 6. Diagram of published predictions of the sunspot number maximum for cycle 23. References with points are those added to the references given by Kane (1997). Black bins are for geomagnetic precursor methods and hatched bins for other precursor methods. Predictions of the present work are labelled with numbers: 1 corresponds to $Ap_{\rm min}$ 2 to Ap_{36}, 3 to $aa_{\rm min}$, 4 to aa_{36}, 5 to NDD, 6 to $aa_{\rm F}$, 7 to Mult 2, 8 to Mult 1, and 9 to $aa^{*}_{\rm max}$.

previous ones, except the prediction by Bravo and Stewart (1997) which is based on coronal hole observations. On the left part of the diagram, it should be noted that the predicted low $RI_{\rm max}$ are frequently obtained with time series analysis. On the right side of the diagram, many of the predictions with high $RI_{\rm max}$ involved the even-odd empirical ‘rule’ predicting higher maximums for the odd numbered cycles. It should be recalled that this rule has, in the past, suffered two exceptions over the ten pairs of observed cycles.

We have studied the skill of methods involving geomagnetic precursors to predict the size of the next cycle. The precursors are: $aa^{*}_{\rm max}$ the late maximum of geomagnetic activity occurring within the last four years of the cycle, $aa_{\rm min}$, and $Ap_{\rm min}$, minimum values of the geomagnetic indices, aa_{36} and Ap_{36}, average values of the indices over the last three years of the cycle, $aa_{\rm F}$, aa index after subtraction of the sunspot cycle as proposed by Feynman (1982) and NDD, number of disturbed days ($Ap \geq 25$) over the entire cycle. The precursors for past cycles have regression coefficients against the maximum of sunspot number, ranging between 0.90 and 0.97. We have shown that most of the methods are improved when bivariate analysis with the minimum of the sunspot number is used as a second independent variable, as shown for $aa_{\rm min}$ and $Ap_{\rm min}$ by previous authors. All regression coefficients, but one, rise to the range 0.94 to 0.97. Finally, with three independent variables, even higher coefficients, close to 0.975, are obtained.

According to our tests, the late maximum of aa index aa^*_{max} appears as one of the best precursors with single-variate as well as with multivariate analysis, and thus, it is not useful to subtract from geomagnetic indices a component proportional to sunspot numbers. This method has the further advantage of predicting the size of the sunspot cycle two years on average before its beginning.

The level of the regression coefficients and the coherency between predictions obtained with different precursors show the relevancy of the concept of the extended cycle. Although geomagnetic precursors appear as the best for prediction purposes, solar precursors measurements are essential for a better understanding of the solar cycle. This is true for those measured during the minimum of the cycle (like polar magnetic fields and high-latitude faculae), but even more important for those measured during the declining phase of the sunspot cycle (coronal holes and large-scale magnetic fields), because they are basic data for the revision of the classical concept of the solar cycle, which appears now more complex than the sunspot cycle alone.

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