PREDICTION OF THE MAXIMUM AMPLITUDE OF SOLAR CYCLES USING THE ASCENDING INFLEXION POINT

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Abstract. To predict solar cycle maximum in terms of smooth sunspot numbers, a method based on the slope at the inflexion point observed during the ascending phase of the cycle is proposed. Application to cycle 23 (beginning in May 1996) gives a predicted value of 103 ± 20 (r.m.s.) for the sunspot number maximum. A comparison with predictions using other methods is given.

1. Introduction

Prediction of the sunspot number cycle is of importance for many applications, including telecommunications and satellite orbitography. Statistical methods like McNish and Lincoln method (McNish and Lincoln, 1949) are widely used on an operational basis (Hildner and Greer, 1990). More recently, methods using neural networks have been tested and give satisfactory results either statistically or on specific problems (McPherson, Conway, and Brown, 1995; Fessant, Pierret, and Lantos, 1996). The McNish–Lincoln method and neural networks are efficient to predict the whole cycle a few months in advance.

Another class of methods, based on observed precursors, predicts the sunspot number maximum years in advance. Geomagnetic activity observed during the declining phase of a cycle or during the minimum is a precursor of the maximum amplitude of the next sunspot cycle, as shown first by Ohl (1968, 1976). A number of authors have developed further use of geomagnetic indices (comparison and references can be found in Lantos and Richard, 1998). The very high correlation coefficients obtained with the different geomagnetic precursors (all above 0.9 and some above 0.97) attests that these methods have intrinsically high reliability, in addition to their contribution to the understanding of the so-called Extended Solar Cycle (see Wilson, 1994). Nevertheless as for the present cycle (with maximum amplitude predicted now in the range 110−200), precursor methods appear to give much too high values (ranging from 137 to 177, according to Lantos and Richard, 1998), it is of interest to consider different methods able to predict the maximum amplitude of a solar cycle before the operational methods mentioned above. We compare here results of a method based on the slope of the sunspot number profile during the ascending phase (Lantos, 1990), with McNish–Lincoln and with neural network predictions.

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TABLE I
Characteristics of solar cycles 9 to 22

Number	Date min	RI min	Date max	RI max	Asc. (month)	Date infl	RI infl	Slope infl
9	July 1843	10.5	Feb. 1848	131.6	55	45	83.0	7.23
10	Dec. 1855	3.2	Feb. 1860	97.9	50	36	75.5	3.75
11	Mar. 1867	5.2	Aug. 1870	140.5	41	31	96.0	7.98
12	Dec. 1878	2.2	Dec. 1883	74.6	60	15	23.9	3.33
13	Feb. 1890	5.0	Jan. 1894	87.9	47	19	46.3	4.03
14	Jan. 1902	2.6	Feb. 1906	64.2	49	17	22.5	2.82
14						31	46.4	2.74
14						40	60.5	2.96
15	July 1913	1.5	Aug. 1917	105.4	49	19	34.7	4.87
16	July 1923	5.6	Apr. 1928	78.1	57	24	47.1	5.27
17	Sep. 1933	3.4	Apr. 1937	119.2	43	24	46.4	4.48
18	Feb. 1944	7.7	May 1947	151.8	39	34	126.2	7.35
19	Apr. 1954	3.4	Mar. 1958	201.3	47	23	109.3	9.97
20	Oct. 1964	9.6	Nov. 1968	110.6	49	22	56.6	6.16
21	June 1976	12.2	Dec. 1979	164.5	42	32	130.9	6.26
22	Sep. 1986	12.3	July 1989	158.5	34	22	104.2	9.99

2. The Ascending Inflexion Point of the Solar Cycle

We consider here the characteristics of a specific point of the ascending profile: the inflexion point. Present work uses the monthly smoothed sunspot number RI_{12}, defined as the moving average of 13 monthly mean values with a weight of $\frac{1}{2}$ for the first and the last values of the series. Monthly sunspot numbers are taken from Sunspot Index Data Center of Brussels Observatory. The maximum of the slope of the ascending profile (computed with a further three-month average) defines the inflexion point. Table I summarises the characteristics of cycles 9 to 22, cycles 1 to 8 being subject to uncertainties because some daily measurements are missing (Waldmeier, 1961). Dates and sunspot numbers RI_{12} for cycle minimum and maximum are given as well as the duration (in months) of the ascending phase. The date (in months after cycle minimum) and sunspot number RI_{12} of the inflexion point is given in columns 7 and 8. Finally, the value of the slope at the inflexion point is given in sunspot number per month.

The inflexion point occurs from 15 to 45 months after the cycle minimum. For most of the cycles, there is no ambiguity in the inflexion date. Nevertheless one of the cycles (number 14) has three inflexion points during the ascending phase with slope values very close to each other. The relation between date and sunspot

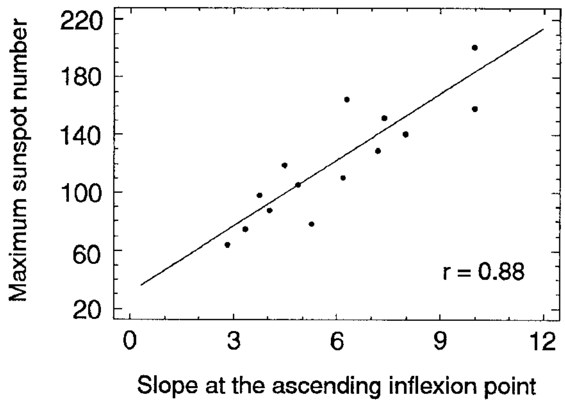

Figure 1. Plot of the maximum amplitude of solar cycles 9 to 22 versus slope at the inflexion point of the ascending profile. The regression line is given.

number of the inflexion point and the same characteristics for the maximum are not strong enough to permit prediction. On the other hand the slope of the ascending profile at the inflexion point and the cycle maximum sunspot number have a correlation coefficient of 0.88 and linear regression analysis gives the relation:

$$RI_{\max} = 15.07\ \mathrm{d}RI/\mathrm{d}t + 30.58\ ,$$

where $\mathrm{d}RI/\mathrm{d}t$ is the slope of the ascending profile taken at the inflexion point, in sunspot number per month. Figure 1 shows the relationship between maximum sunspot number of the cycle and the slope at the inflexion point. The standard error of estimate of the linear regression is 19.7. Note for comparison that the well-known anticorrelation between ascending phase duration and maximum sunspot number (which can not be used for prediction because the first is known when the second has occurred) gives a correlation coefficient of only 0.61 in module.

3. Application to Cycle 23 and Discussion

For cycle 23 the inflexion point occurred in February 1998. The slope at the inflexion point was 4.78 and thus the prediction for the RI_{12} maximum (according to the formula given above) was 103 ± 20 (r.m.s.). In fact this prediction was available in November 1998, tacking into account the six months needed to get the smoothed sunspot number of February and the two further months needed to smooth the slope and to ensure that the maximum of the slope was past. At that time, the McNish and Lincoln method, one year in advance, was giving for October 1999 a value of 144 with the NOAA Boulder implementation (*Solar Geophysical Data*, 1998) and 143 with the Meudon implementation. The neural network method developed by CNET-Lannion, in use at Meudon Warning Center, was predicting 151 for October 1999 and for the same month, specific methods used at Brussels SIDC were

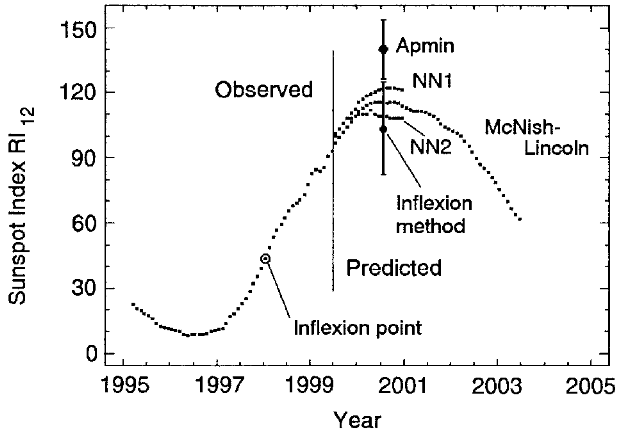

Figure 2. Observed and predicted values of the smoothed sunspot number for cycle 23. McNish–Lincoln and neural network predictions (NN1 and NN2) are given with small squares. Result of the prediction using geomagnetic index *Ap* during the past minimum is labelled Apmin. Result of the inflexion point method is given as well as location, in 1998, of the inflexion point itself. Error bars correspond to standard deviation.

predicting 157 and 148 (*Sunspot Index Data Center Circular*, 1998). Those values were lowest estimates of the cycle maximum amplitude, as October 1999 was still in the ascending phase.

If now we compare the prediction of the inflexion point method with more recent predictions (obtained with data available in January 2000), we obtain results shown in Figure 2. The small squares are, on the left, the observed RI_{12} and, on the right, the predicted one. In addition to the results of a McNish and Lincoln method implemented at Meudon Observatory by R. Chopinet, those of two neural network methods, with different architecture are given : NN1 is a method already mentioned, developed by F. Fessant and X. Lamming at CNET-Lannion (Fessant and Lamming, 1997), NN2 is a method developed by Y. Landrot at Meudon Observatory. SIDC-Brussels and NOAA predictions are similar (*Sunspot Index Data Center Circular*, 2000; ITU, 2000). Most of the operational methods are giving (in January 2000) a date of the maximum during the first semester of 2000, or at the beginning of the fall.

The label ‘Apmin’ indicates the prediction made with *Ap* geomagnetic index during the past minimum of the cycle (Lantos and Richard, 1998). Among the different methods using geomagnetic indices, this method was giving the lowest of the predictions. The result of the inflexion point method is also given on Figure 2, as well as the inflexion point location, on the left side of the figure. Error bars correspond to standard deviation. Thus geomagnetic precursor predictions are only very marginally compatible with the most probable sunspot maximum value of cycle 23, while the result of the inflexion point prediction appears as quite satisfactory.

4. Conclusion

The method using the inflexion point of the ascending phase of the cycle has given an early estimate compatible with the values to which the operational methods are now converging. In 1998, the slow rise of the sunspot numbers was already an indication of the moderate amplitude of the cycle 23, at a time where the other methods were predicting too high values. This fact does not mean that the other methods are less efficient in general. The McNish–Lincoln and neural networks are to be restricted to predictions about six months in advance only during the ascending phase to obtain reasonable precision (Hildner and Greer, 1990). The methods based on geomagnetic activity are still the best available for very early predictions, because all geomagnetic predictors give correlation coefficients above 0.9. Nevertheless, the success of these methods during the past cycles must not slow down research on alternative methods. This is what the present cycle is teaching us.

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