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SOLAR CYCLE PREDICTION: COMBINING PRECURSOR METHODS WITH MCNISH AND LINCOLN TECHNIQUE

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Abstract :
Medium-term and long-term prediction of the magnitude of the maximum of smoothed sunspot numbers, and thus of the solar cycle time profile, is a basic input for many space environment predictions. The widely used statistical technique of McNish and Lincoln is systematically compared to predictions based on precursors, either related to the cycle time profile characteristics or to geomagnetic indices. It is shown that when cycles 13 to 23 are considered, all prediction methods give, at least for one of the cycles, an error much larger than 20 %, an inadequate result. None of the methods is fully reliable. Thus it is proposed to combine the predictions based on precursors and to improve McNish and Lincoln results with them in order to limit such rare but large errors and to improve significantly the reliability of the predictions performed in the course of the solar cycle ascending phase.

1. Introduction

Predictions of the cycle of sunspot number RI_{12} (Waldmeier, 1961) are of prime importance for most of the space weather applications. Prediction of the solar cycle is strongly needed for telecommunications and satellite orbitography because the solar flux in the UV, EUV and X-ray ranges, which is partly emitted in magnetic loops located above solar active regions, is highly correlated to the sunspot number. Radiation belt characteristics (and their impact on satellite electronics) are also solar cycle dependent, as well as atmospheric chemistry (and its impact on satellite surfaces). Because solar flare and coronal mass ejection frequencies are closely related to the sunspot cycle, applications involving solar particles (radiation dose received by astronauts and by satellite electronics for example) are demanding long term sunspot cycle predictions. Finally solar sunspot cycle prediction is used to predict cosmic ray intensity and radiation doses received by air crews (Lantos, 2005), because of the galactic cosmic ray modulation by heliospheric magnetic field.

For medium-term solar cycle prediction (i.e. predictions months in advance), the statistical method of McNish and Lincoln (M&L) remains the reference (McNish and Lincoln, 1949, Steward and Ostrow, 1970; Greer, 1993). Indeed, the M&L method is the only cycle prediction method widely used for operational purposes. Among implementations one can mention: ESA European Space Operations Center (Darmstadt, Germany), British Geological Survey (Edinburgh, UK), NOAA National Geophysical Data Center (Boulder, USA), NASA-MSFC (Huntsville, USA) or Paris-Meudon Observatory Warning Center (for satellite orbitography for CNES, at CLS[1], Toulouse, France and, for ESA, at GFZ[2], Potsdam, Germany). The formula published by McNish and Lincoln in 1949 is for predicting the annual sunspot numbers of the current solar cycle. A first approximation to the prediction of a future value in a cycle is the mean of all past values for that part of the cycle. This estimate can be

[1] Collecte Localisation Satellite, Toulouse, France, a CNES subsidiary
[2] GeoForschungsZentrum, Potsdam, Germany

improved by adding to the mean a correction factor proportional to the departure of earlier values to the cycle from the mean cycle. The correction factors are determined by the method of least squares. This method has been greatly improved by Stewart and Ostrow in 1970. Indeed, they have described an adaptation of the M&L technique to sunspot data spaced at monthly intervals, which allows the prediction of monthly mean values. All M&L techniques now in use predict the time profile of RI_{12}, the monthly smoothed sunspot number for the current solar cycle. The prediction could be updated each month using the last observations.

An evaluation of the skill of the method has been published by Hildner and Greer (1990). They have confirmed that the method is well adapted for predicting solar cycle a few months in advance. Six months of M&L predictions are generally issued for operational purposes. For longer term predictions, the results are disappointing at the beginning of a new cycle, because the RI_{12} cycle profile is not well-established before about half of the ascending phase. The declining phase is well predicted by the M&L technique (Fessant, Pierret, and Lantos, 1996), when the observed solar cycle maximum value is taken as the starting point (Greer, 1993), as is done in most implementations. When the evaluation of the M&L method by Hildner and Greer (1990) is compared with the present work, there are some subtle differences. Indeed Hildner and Greer call "month of prediction" the month of the last value of RI_{12} available. Here we consider throughout the paper (except otherwise specified) the month at which RI_{12} and the corresponding predictions are actually available, six months later.

More recently, non-linear statistical methods using neural networks have been tested and give satisfactory results either statistically or on specific problems. They have been compared to the M&L technique (Macpherson, Conway, and Brown, 1995, Fessant, Pierret, and Lantos, 1996). Unlike the M&L technique, for which different implementations give similar results, neural network results are dependent upon chosen network architecture and of data used for learning. Thus they are almost impossible to duplicate identically, so as detailed comparison between them remains difficult. Waldmeier (1968) has proposed a method to predict solar cycle profiles from the steepness of the ascending phase of the cycle. It was originally based on graphic time profiles of the cycles, and after being computerised, it is in use as a standard method at SIDC[3] (Brussels Observatory). Hathaway, Wilson, and Reichmann (1994) have proposed a medium-term method of prediction based on the same empirical ground, but using cycle profiles simulated with an analytic function using two parameters (hereafter called the HWR function). The two parameters are the date of beginning of the solar cycle and its amplitude. The prediction is done by fitting the past sunspot number observations to that of the given cycle. In order to advance the epoch at which the solar cycle profile could be correctly predicted, both Denkmayr and Cugnon (1997) at SIDC and Hathaway, Wilson, and Reichmann (1999) have proposed to combine precursors (see next paragraph) with their own prediction methods.

A number of other different techniques have been proposed to predict specifically the magnitude of the solar cycle maximum, RI_{max}, in most cases years in advance (long-term predictions). We consider here only the best precursor methods for which the correlation coefficient is above 0.8. A first class of precursors is deduced from the characteristics of the observed RI_{12} time profile. Indeed, it has been shown that the maximum amplitude of a solar cycle is correlated to the steepness of the ascending phase of the cycle. A second class of precursors is based on geomagnetic activity measurements. As shown first by A.I. Ohl (Ohl, 1966; Ohl, 1976), geomagnetic activity observed during the declining phase of a cycle or

[3] Sunspot Index Data Center, now renamed Solar Influence Data analysis Center, Brussels, Belgium

during the minimum is a precursor of the maximum amplitude, RI_{max}, of the next sunspot cycle. Many authors have developed further use of geomagnetic indices (see references in Lantos and Richard, 1998). The very high correlation coefficients obtained with different geomagnetic precursors attest that these methods have intrinsically high reliability and confirm the relevance of the 'extended solar cycle' paradigm (see Wilson, 1994). A systematic evaluation of a number of geomagnetic precursor techniques can be found in Lantos and Richard (1998).

The purpose of the present work is to evaluate the M&L method for cycles 13 to 23, to compare the results with those of the precursor methods, and to propose improvement of the use of the M&L technique thanks to a combination with the best of the precursors. Indeed during the first years of a new cycle, all the medium-term prediction methods, including the M&L method, have difficulties to predict properly, more than a few months in advance, the time profile of the cycle and the solar cycle maximum amplitude. The combination with precursors extends the range of application of the M&L method. In addition, it will be shown that the M&L method, as well as all the precursors, have rare but unacceptable errors of prediction, much above 20 %. As those errors could be prejudicial even if they concern very few of the cycles, none of the methods could be considered as fully reliable. The combination limits the errors, compared to those of each of the methods. More generally the combination statistically improves the results of the methods. Finally the use of precursors obviously permits an earlier prediction of the cycle maximum and thus of the time profile of a cycle. As some of the precursors are available during the declining phase of the previous cycle, the prediction is available even before the beginning of the cycle. Indeed Waldmeier's method or the HWR function could be used to predict the time profiles of the coming cycle from the prediction of RI_{max}.

2. Comparison of Methods of Prediction for Solar Cycle 23

The present cycle 23 is a good example because a number of methods of predictions are now in operation. The maximum of sunspot number RI_{12} for cycle 23 equals 120.7, in April 2000. In fact because of smoothing and of necessity to detect the extremum, the result only becomes available seven months later, in November 2000. The right-hand-side panel of Figure 1 compares the predictions versus time for three medium-term methods, from October 1998 to September

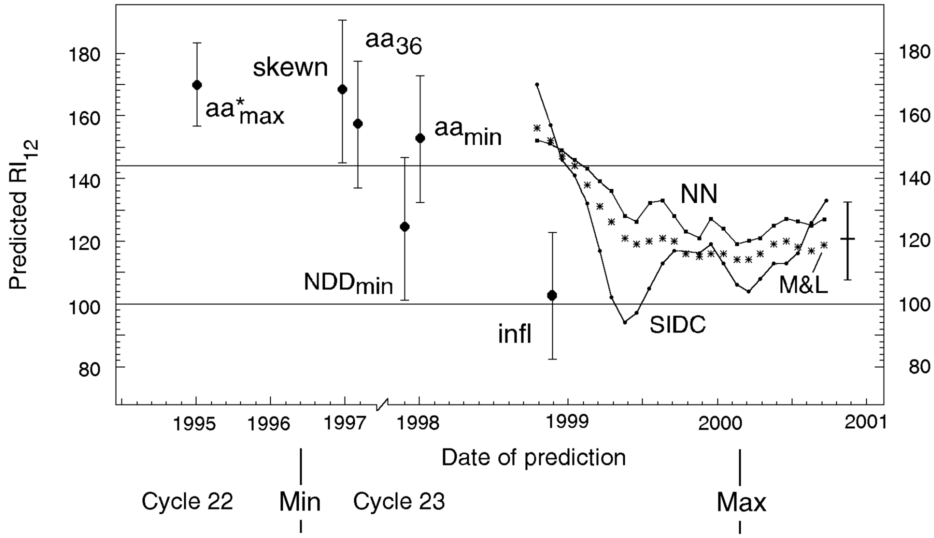

Figure 1: Evolution of the predicted maximum amplitude for cycle 23 as a function of the actual date of prediction. On the right, medium-term predictions are plotted from October 1998 to September 2000 and labelled M&L, NN and SIDC. On the left, predictions based on precursors are indicated (see text). The observed maximum of cycle 23 is 120.7, in April 2000, and this value of RI_{12} is available seven months later, in November 2000. It is indicated on the right side of the figure, with a cross also giving the range of ± 10 %. The region between horizontal lines indicates the range ± 20 % around the observed RI_{12} maximum.

2000. The standard predictions of SIDC (SIDC, 1998-2000) are based on Waldmeier's (1968) analysis of RI_{12} time profiles. A second method is a neural network (Fessant and Lamming, 1997), labelled NN. The third method is the M&L method. The considered neural network and M&L methods are those as implemented at Meudon Observatory. The M&L method gives predictions over four years. It is indicated with asterisks. At the end of 1998 (at about half of the ascending phase of the cycle), all three medium-term methods give prediction which falls in the range ± 20 % of the observed maximum (region between horizontal lines). Starting in May 1999, both the M&L and NN methods converge and stay in the range ± 10 % of the final result (a range indicated with a cross located at the time where RI_{max} becomes available).

For cycle 23, the results of medium-term methods are compared in Figure 1 to those of precursor methods. All the precursor methods are the result, over the different cycles, of linear regressions where the RI_{12} maximum amplitude, RI_{max}, is in general the dependent variable. The regressions do not include the precursor needed to predict cycle 23. In Figure 1, the error bars give the standard error of estimate of the methods. They are located at the time where the prediction becomes available. Let us consider some of the precursor methods in their usual chronological order:

1- [aa_{max}*] A first method, noted aa_{max}* (Lantos and Richard, 1998), is using aa antipodal geomagnetic indices. This geomagnetic index, based on antipodal measurements of the geomagnetic field (stations of Canberra in Australia and Hartland in U.K.), has been calculated back to 1868 by Mayaud (1980). The precursor is the value of a late maximum observed during the last four years of the cycle. This maximum is related to to the recurrent geomagnetic activity due to a coronal hole extension (Svalgaard 1977; Legrand and Simon, 1981). The regression analysis is done with RI_{max} and Aa_{max}* observed at the end of cycles 12

(1878-1890) to 21 (1976-1986), and the precursors needed to predict cycles 13 to 22.
The regression gives:

$$RI_{max} = 7.559 \times Aa_{max}^{*} - 55.51.$$

The predicted value is 169.8 for cycle 23. Evaluation of the method shows that with a correlation coefficient of 0.96 and a standard error of estimate of 12.8, it is one of the best methods. In addition it is available well before the end of the previous cycle. Indeed the late maximum occurs in average 2 years and 4 months before the end of the cycle.

2- [skewness] A second method is based on a work published by Ramaswany (1977) which shows that the skewness of a given cycle is a precursor of the maximum of the following one cycle. The skewness γ, a classical parameter for asymmetry of distributions in statistics, is defined as :

$$\gamma = \mu_3/\sigma^3 = \mu_3/\mu_2^{3/2}$$

where μ_3 is the third moment about the mean, σ is the standard deviation and μ_2 is the second moment about the mean (variance). The linear regression is applied to the ratio **R** of the maximum of the following cycle to the maximum of the given cycle, the skewness being the independent variable. A modified method (Lantos, 2006) with a separation of odd- and even-numbered cycles is used here. It is here calculated with the time profiles of RI_{12} of different cycles. The linear regression formulae are:

$$\mathbf{R} = -2.1092\,\gamma + 1.9418 \text{ when } \gamma \text{ corresponds to even-numbered cycles and}$$

$$\mathbf{R} = -1.2552\,\gamma + 1.3570 \text{ when } \gamma \text{ corresponds to odd-numbered cycles .}$$

When the observed cycle is even-numbered, the regression coefficient equals – 0.857 and when the observed cycle is odd-numbered the regression coefficient equals – 0.831. The standard error of estimate is 22.1. For the prediction of cycle 23, the skewness of cycle 22 is found to be 0.419.

3- [Aa_{36}] A method suggested by Ohl (1976) considers the average geomagnetic activity during the last three years of the cycle. This precursor is noted here Aa_{36}. According to Lantos and Richard (1998), the regression gives:

$$RI_{max} = 6.012 \times Aa_{36} + 9.85$$

For cycle 23 the predicted value is 156.7. The correlation coefficient between RI_{max} and Aa_{36} equals 0.91 and the standard error of estimate is 19.4. The prediction is available when the date of the RI_{12} minimum is known (i.e. seven months after the cycle minimum).

4- [Aa_{min}] Another method suggested by Ohl (1966), involves the linear regression of RI_{max} (dependent variable) with Aa_{min} (independent variable), the minimum of the geomagnetic aa index, smoothed over twelve months as done for RI_{12}. It is generally observed six months to one year after the cycle minimum. The regression (Lantos and Richard,1998) gives:

$$RI_{max} = 8.409 \times Aa_{min} + 12.85$$

The prediction for cycle 23 is 153.4. Evaluation shows that the correlation coefficient equals 0.90 and the standard error of estimate is 19.9, when solar cycles 12 to 22 (1986-1996) are considered.

5- [NDD_{min}] A method based on the number of geomagnetically disturbed days (NDD) has been suggested by Wilson (1990). The days are considered as disturbed when the Ap planetary index (Bartels, 1949) is larger than 25. The proposed precursor NDD_{min} is the minimum, over a cycle, of NDD. The linear regression is between RI_{max} (dependent variable) and NDD_{min} (independent variable). The minimum occurs at about the same time as the minimum of the geomagnetic index of the previous method. Here the number of disturbed days for each month is smoothed as done for RI_{12}. The Ap index is available only since 1932, but it could be extended to 1868 (see for example Lantos and Richard, 1998), using the aa antipodal index. Thus using cycles 12 to 22, the linear regression between the predicted RI_{max} and the precursor NDD_{min} shows a correlation coefficient equal to 0.86 and a standard error of estimate of 22.7. The regression gives the relation:

$$RI_{max} = 46.72 \times NDD_{min} + 77.12.$$

The prediction for cycle23 is 123.85.

6- [inflexion point] Finally we select a method based on the slope at the inflexion point encountered during the ascending phase of the cycle (Lantos, 2000). The linear regression is between the predicted RI_{max} and the slope of the ascending RI_{12} time profile at the inflexion point (independent variable). For solar cycles 9 (1843-1855) to 22 the correlation coefficient is found to be 0.88 and the standard error of estimate is 19.7. The inflexion point is observed on average about 20 months before the solar maximum, but the advance could vary from 5 months (cycle 18 from 1944 to 1954) to 45 months (cycle 12 , a cycle with a long ascending phase of 60 months). The regression gives:

$$RI_{max} = 15.07 \times dRI/dt + 30.58,$$

with the precursor equals 4.78 for cycle 23.

For cycle 23, the four selected precursor methods based on geomagnetic indices are predicting: 170 ± 13 (r.m.s.) with Aa_{max}*, 157 ± 20 with Aa_{36}, 153 ± 20 with Aa_{min} and 124 ± 23 for NDD_{min}. The two precursor methods based on characteristics of the cycle time profile give 103 ± 20 with the method based on inflexion point and finally 168 ± 22 with the method involving the skewness of sunspot number cycles. Predictions are to be compared to the observed solar cycle maximum of 120.7. This shows that even the best precursor methods give sometimes very unacceptable predictions. Indeed Aa_{max}* shows an error of 41 % (the worst case for the prediction of cycles 13 to 23). The same is true for cycle 23 with the predictions using the skewness of the previous cycle. More generally it will be shown later that unacceptable error exists, at least for one cycle, whatever the precursor considered. This may justify the search for a combination of methods limiting such large errors. Combination of methods will also improve statistics and thus precision of the predictions. In addition Figure 1 shows that three predictions based on geomagnetic activity: Aa_{max}*, Aa_{36} and Aa_{min} are outside the error range of ± 20 %. The poor quality of the predictions of those precursors is specific to cycle 23 (see section 3). Cycle 22, for example, was given worse predictions by the M&L method and much better predictions by geomagnetic precursors. This shows that evaluation and comparison of prediction methods must involve as much solar cycles as possible.

3. Precursor and M&L Predictions for Cycles 13 to 23

Figure 2 shows, for each cycle from cycle 14 (1902-1913) to the present cycle 23, the predictions of the maximum value of the smoothed sunspot number RI_{12} as a function of the date at which the prevision could be made. The open diamond, on the right side of each frame, is the observed value of the RI_{12} maximum, RI_{max}. It is available seven months after the actual date of cycle maximum which is indicated with a dashed line. The actual RI_{12} cycle minimum date is similarly indicated on the left side. The range between horizontal lines corresponds to ± 10 % variation around the observed value of the RI_{12} maximum. The string of points corresponds to the monthly predictions for the maximum of RI_{12} obtained with the M&L technique (as implemented at Paris-Meudon Observatory Warning Centre). The string starts when the minimum of the cycle could be actually detected. Predictions with precursor methods (except method 1) are indicated in Figure 2 approximately at the times at which each of them is available. The precursor corresponding to a cycle to be predicted is suppressed from the corresponding linear regression. Symbols of the different precursors are given within the frame of cycle 14.

With respect to the M&L technique, comparison of the different frames of Figure 2 shows that the results are converging to the observed value of RI_{max}. Nevertheless the shape of convergence varies from one cycle to another. If we consider the last entry into the 10 % range (range indicated by horizontal lines), this precision is reached very early (3.5 years before the maximum) for cycle 20 and two years in advance for only three other cycles (namely cycles 16, 19 and 23). Note that two years before the maximum corresponds, on average, to the middle of the ascending phase. For six other cycles, the precision of 10 % is reached less than eight months before the maximum. When the range ± 20 % is considered, the distribution is reversed: only three out of the ten cycles (namely cycles 15, 17 and 22) reach the requested precision less than eight months in advance, while three others do the same one year in advance and the maximum of the rest is predicted with a precision of 20 % more than two years in advance. Figure 2 shows that cycle 20 is very exceptional with respect to the M&L predictions. Indeed the prediction remains in the range ± 20 % since the beginning of the cycle and in the range ± 10 % during the 42 months before the maximum of the cycle. In contrast, for cycle 22 RI_{max} was overestimated strongly up to a date very close to the maximum. This exceptional case has been discussed by Hildner and Greer (1990) and by Fessant, Pierret, and Lantos (1996). Hildner and Greer (1990) have shown, according to their last figure, that error of prediction was 40 % one year in advance, 30 % six months in advance and the error of the prediction was still 17 % at the time where the actual maximum amplitude could be finally calculated.

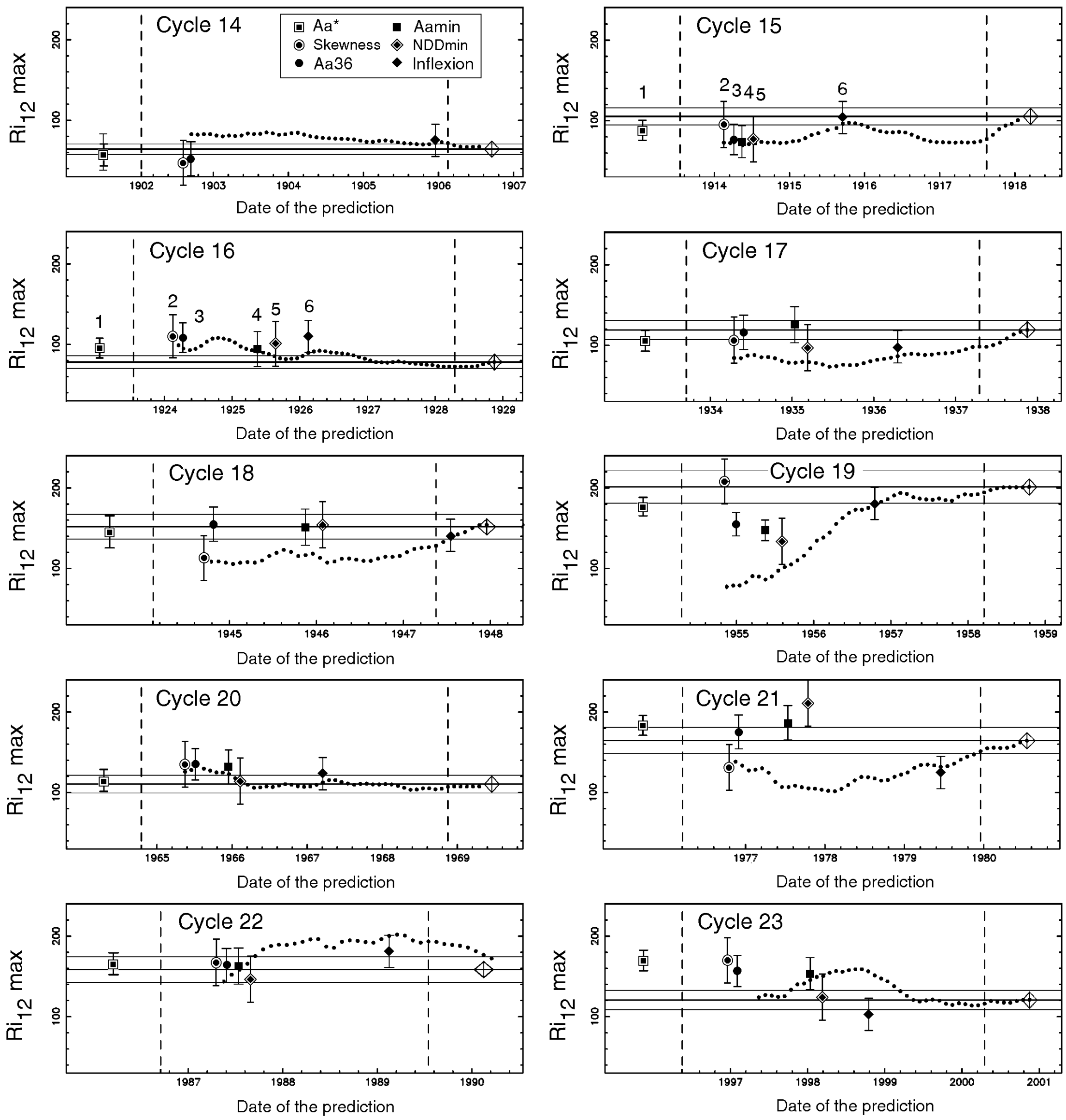

Figure 2: Evolution of the cycle maximum predicted with the M&L technique (points) and with selected precursors. Symbols corresponding to different precursors are given in the frame of cycle 14 and explanation of the methods is in the text. The symbols are located at the date at which the prediction is available, and for cycles 15 and 16 the number of the method (see section 2) is explicitly given. The diamond indicates the date of availability and the observed maximum value of the cycle RI_{max}. Vertical dashed lines give actual cycle minimum epoch and actual cycle maximum epoch. Regions between horizontal lines correspond to the interval ± 10 % around the observed maximum.

Another remark about Figure 2 is that the performance of the M&L prediction is not a function of the amplitude of the cycle: some cycles, like cycles 15 (RI_{max} = 105) or cycle 17 (RI_{max} = 119), while close to the reference average of the past cycles, are not better predicted than the others.

As mentioned above for cycle 22, the M&L method could give unacceptable results with an error of prediction of 40 % one year in advance. The same occurs with precursors. Indeed Figure 2 shows that at least one of the errors on prediction is much too high, even with the most reliable methods. This is true for all the precursors considered here. Indeed the error in the worst case is 30 % for Aa_{min} (cycle 15) and NDD_{min} (cycle 14). It is 40 % for Aa_{36} (cycle 16), Aa_{max}* (cycle 23) and skewness (cycle 23). For the method based on inflexion points, the error is 45% (cycle 16). For applications, those errors could be prejudicial even if they occur on very few of the cycles. This is an important point, which limits strongly the use of individual precursors for operational purposes and tends to accredit unjustified doubts on the reliability of the prediction methods. The suppression of the very large errors on a given cycle is even more important that the search for better statistical precision.

The most synthetic approach for a comparison between precursor and M&L predictions is derived from a diagram proposed by Hildner and Greer (1990) for evaluation of the M&L results. Figure 3 gives the percentage of cycles for which predictions are better than 10 % and 20 %, as a function of the month of the cycle at which the prediction is done. As in the Hildner and Greer diagram, the "error bars" show how the results would have differed if one more or one fewer cycle met the accuracy criterion. In addition to the M&L squares (for 20 % accuracy) and points (for 10 % accuracy), symbols similar to those of Figure 3 give the performance of the five precursor methods. For each, the upper symbol is for accuracy better than 20 % and the lower symbol for accuracy better than 10 %.

On the one hand, with the precursor methods the reliability on the precision of the prediction is limited because only 25 to 50 % of the cycles are predicted with a precision better than 10 %. Nevertheless this is available during the descending phase of the previous cycle, or early in the ascending phase of the predicted cycle, much before the prediction with the M&L technique, which is better than 25 % only two years after the beginning of the cycle. On the other hand, if a lower precision of 20 % is accepted, some of the precursor methods (like Aa_{max}*) appear as reliable because up to 82 % (9 over 11) cycles are predicted. As shown in Figure 3, those predictions are available much in advance over the M&L method which obtains the same reliability more than three years after the beginning of the cycle.

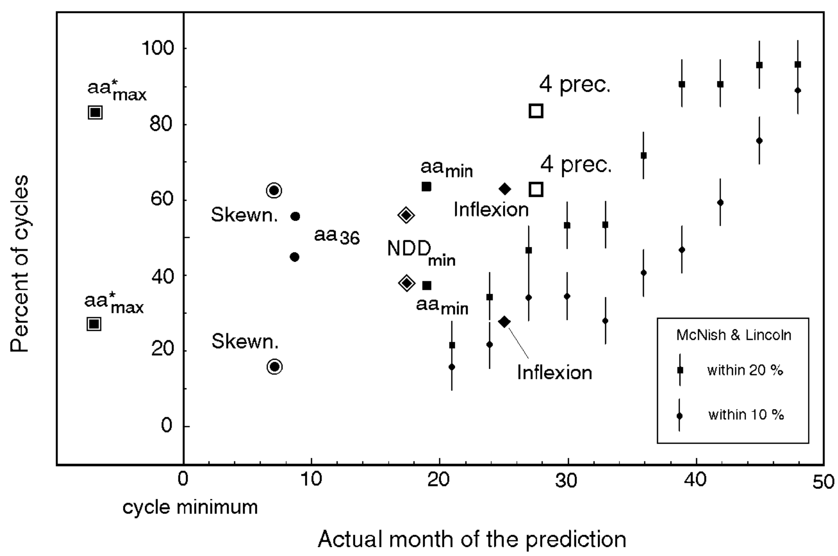

Figure 3: Statistical accuracy of the prediction of the maximum amplitude of the cycles as a function of the average actual month of the prediction. For the M&L method, the squares show the percentage of the cycles for which its prediction was accurate with ± 20 %. Similarly the points show the same for accuracy of ± 10 %. The two symbols corresponding to each precursor method give the same information (upper symbol for ± 20 %, lower symbol for ± 10 %). The "error bars" show how the results would have differed if one more or one fewer cycle met the accuracy criterion. Open squares are for the weighted mean of the four precursors selected to be combined with the M&L method (see Section 5).

5. Improvement of McNish and Lincoln Predictions with Precursor Results

In order to avoid rare but large errors and to improve significantly the reliability of the predictions performed in the course of the solar cycle ascending phase, it is proposed to combine the predictions based on precursors and to improve the M&L results with them. The method uses a weighted mean of the results of a few selected precursors and of the results of the M&L technique:

$$\Sigma w_i \, P_i \,/\Sigma w_i$$

where w_i are the weights and P_i are the predictions. The weights are inversely proportional to the variance (square of the standard deviation). The number of terms varies with the epoch depending on the predictions available. The maximum RI_{max} of the coming cycle is given by precursors very early and improved when a new precursor becomes available. Before the beginning of a new cycle and thus before the start of the M&L method, Waldmeier's method or the HWR function could be used to predict the time profiles of the coming cycle from the prediction of RI_{max}. To select the precursors, two conditions are to be fulfilled. The first one is a correlation coefficient sufficient to ensure the reliability of the method. We have limited the survey to methods with correlation coefficients above 0.8. The second condition to combine methods is their independence. Hence, among the precursors studied before, two (namely Aa_{36} et Aa_{min}) will not be used anymore because their errors are highly correlated to the errors of our reference method Aa_{max}*. Indeed the correlation coefficient with Aa_{max}* is 0.814 for Aa_{36} and 0.852 for Aa_{min}. For the same reason, the methods of Thompson (1993), Feynman (1982) and Kataja (1986) have not been considered here.

The selected precursors are finally (a) Aa_{max}*, (b) the skewness of the previous cycle, (c) the minimum of NDD and (d) the slope of the ascending phase at the inflexion point. Methods (a) and (b) are available before the end of the previous cycle (see Lantos, 2006 for method (b)). Method (c) is available about one year after the minimum and method (d) about two years after the minimum (see figure 2). As mentioned above, Figure 2 shows, for method (1) which is the best of our precursors, that for nine out of eleven cycles, the maximum RI_{max} is predicted with a precision better than 20 %. But the maximum RI_{max} is predicted with a precision better than 10 % for only three cycles. The weighted mean of the four precursors, available at the same time as the inflexion point, gives the same number of cycles predicted with precision better than 20 % and, more revealing, seven cycles with precision better than 10 %. For comparison with the result of individual precursors, the performance of the combination of the four precursors is indicated (open squares) in Figure 3. The weighted mean of the precursors also reduces the error on the prediction of RI_{max} for cycle 16 from 45 % (worst case with inflexion point method) to 27 %. Similarly the worst cases of precursors Aa_{max}* and skewness (respectively in error by 41 % and 39 %), encountered for prediction of cycle 23 are reduced to 23 % by the combination of the precursors.

The M&L method has a standard error of estimate decreasing when the maximum approaches. Figure 4a, adapted from Hildner and Greer (1990) and considering the M&L estimates of RI_{max} during cycle 22, shows that at the beginning of a cycle, the standard error of estimate is much larger than those of the precursor methods (which are between 12 and 23 according to section 2). Thus the weight of the M&L results is very small compared to the weight of the precursors. The standard error of estimate of the M&L method becomes much similar to those of the precursor methods 30 months after the minimum of the cycle. Close to the maximum of the cycle, the standard error of estimate of the M&L method becomes smaller than those of the precursors. Thus the prediction resulting from the weighted

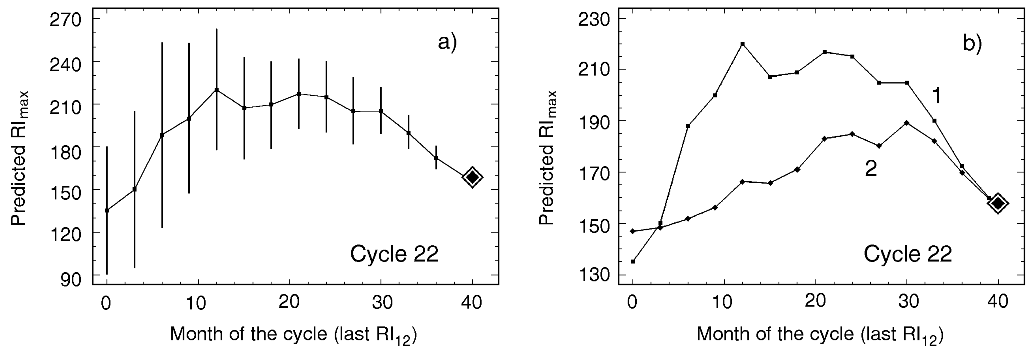

Figure 4: a) Hildner and Greer estimates of RI_{max} for cycle 22 as a function of the month of the last RI_{12} used for the prediction. The bars are the standard error of estimate (in place of the 90 % confidence range of the original figure of Hildner and Greer). The observed RI_{max} is indicated with a diamond.

b) Improvement of the prediction when the combination with precursors is applied to the predictions of the M&L method. Curve 1 is the result of the M&L method alone as in Figure 4a. Curve 2 is the result after combination.

means, dominated by the M&L estimates, converges to the observed value of the maximum RI_{max}, as the M&L prediction does. Figure 4b shows, for cycle 22, the comparison of the Hildner and Greer predictions (curve 1) with the predictions after improvement with precursors (curve 2). Cycle 22, which is particularly badly predicted with the M&L method (error of about 40 %), remains in the range ± 20 % after improvement.

Table I Maximum errors of the M&L predictions of RI_{max} during the ascending phase of the cycles.

Predicted cycle	Max. error with non-improved M&L method %	Max. error with improved M&L method %
13	41.5	11.9
14	32.3	10.3
15	-33.8	-26.3
16	38.4	27.5
17	-38.5	-17.7
18	-30.4	-18.3
19	-61.5	-15.4
20	17.4	8.1
21	-38.1	-4.8
22	27.4	13.8
23	31.6	39.8

Table 1 gives for cycles 13 to 23 the maximum errors in percentage when the non-improved and the improved M&L methods are used to predict the amplitude of the cycle maximum during the ascending phase of the cycles. The range is taken from the beginning of the predictions (see Figure 2) to the maximum itself. The M&L estimates are according to the Paris-Meudon implementation. Figure 5 summarises the same comparison in absolute scale. To each cycle are assigned two bars. The bar on the left represents the range of variation of the prediction with the M&L method alone, from the beginning of the cycle to the cycle maximum. The bar on the right is the same for the M&L predictions improved with precursor methods. Figure 5 and Table 1 show that with the M&L method alone, nine out of eleven cycles show errors larger than ± 30 %. The worst case is with cycle 19, which, at the beginning of the cycle is predicted with an error of 62 % (see also Figure 2). After improvement by combination with the precursors, the range of variation is reduced, for cycle 19, to less than 15 %. and eight out of eleven cycles have predictions in the range ± 20 % during the ascending phase of the cycle. Out of eleven cycles, ten have been improved by the combination method, sometimes very appreciably, like in the case of cycles 19 and 21 (for which the range of variation is respectively reduced by a factor 4 and 8). The only case with degradation is for cycle 23 (the range is 1.25 times larger than with the M&L method alone). Indeed the two precursors available at the beginning of the cycle (namely Aa_{max}* and skewness) have unfortunately both their worst prediction (errors of about 40 %) for this cycle. The estimate is somewhat corrected later by the two other precursors. When the prediction with the inflexion point becomes available, the four combined precursors give a prediction still in error of 24 %. Finally the M&L method forces the convergence to the observed value, showing the interest to use the M&L method in addition to the precursors to predict the maximum of the cycle. As the time profile of a cycle is closely related to the amplitude of the maximum RI_{max} (Waldmeier, 1968), an improved prediction of the maximum means a better prediction of the time profile which is obtained from the standard application of the M&L method. It is likely that the M&L method, limited presently to six months for operational applications could be used to extend its horizon of prediction to few years when the M&L method is combined with precursors as proposed here.

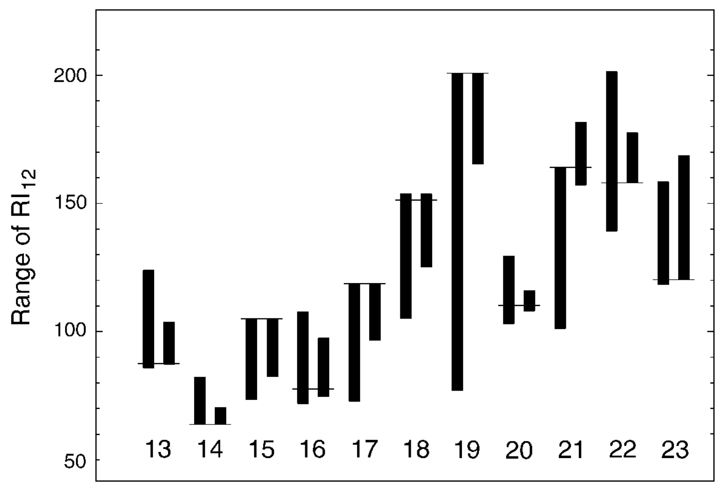

Figure 5: Ranges (bars in black) over which the expected amplitudes of the maxima of the cycles 13 to 23 are located, as predicted from the minimum to the maximum of each cycle. For each cycle the first bar is with the M&L method alone and the second shows improvement when combination with the precursors (by weighted mean) is applied. The horizontal line shows the observed cycle maximum value.

6. Conclusion

A first obvious remark is that in contrast to what is sometimes assumed when different methods are compared (section 14.5 of Wilson, 1994), a good or a bad prediction of the current cycle is not sufficient to evaluate properly the performance of a prediction method. The only realistic approach is with statistical evaluation involving as much cycles as possible. Here the comparison of the McNish and Lincoln (M&L) method and of precursor methods has involved eleven cycles, for which the geomagnetic index aa is available.

The present paper has analysed the advantages and the limitations of the M&L method and of the precursor methods to predict the amplitude of the maximum of sunspot index RI_{12} and thus the time profile of the cycle. On the one hand, the M&L method is limited in precision during the ascending phase of the cycle. In fact it is mostly used with a prediction horizon of six months only. Nevertheless the M&L method could be updated at each month and it converges to the actual maximum of the cycle. On the other hand, the precursors are available long time before, but give sometimes predictions very far from the actual maximum amplitudes. A sequence of methods has been proposed, each new precursor being introduced when available in a weighted mean including previous precursors as well as the M&L results. By doing this, an optimised prediction of the next cycle maximum is available continuously. In addition to the improvement of the statistics, the combination of different methods has the advantage to give very early predictions, much in advance compared to the M&L method alone, and to limit unacceptable predictions sometimes encountered when using a single precursor method or the M&L method alone. With the proposed combination, the M&L method, which was originally a medium-term prediction scheme, becomes a long-term method and is able to predict, with acceptable precision, the sunspot cycle years in advance.

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