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THE SKEWNESS OF A SOLAR CYCLE AS A PRECURSOR OF THE AMPLITUDE OF THE NEXT

P. Lantos
Observatoire de Paris

Abstract

As a precursor for predicting the maximum amplitude of the coming solar cycle, the skewness of the previous cycle proposed by Ramaswany in 1977 is revisited. The reliability of the prediction method is improved by separating odd and even cycles. A first method is proposed on the basis of calculated skewness. In that case the prediction is available at the end of the previous cycle. A possibility to anticipate the availability of the skewness by about one year is pointed out. A second method, adding prediction of the skewness itself is studied. The statistical reliability is lower than in the first case, but the prediction of a cycle maximum is available at the maximum of the previous cycle.

1. Introduction

The prediction of solar activity is of importance for many applications. Space technology needs its prediction to estimate the orbit, lifetime, and necessary amount of shielding of satellites. Manned space flights in high-inclination orbits or in trajectory outside the protection of the Earth's magnetic field need close monitoring as well as prediction of solar activity. Radio communications, electric power and cable network operations and pipeline maintenance could also be disturbed by solar activity (Lanzerotti, 1983; Lantos, 1997). The prediction of the solar cycle, using the smoothed sunspot index RI_{12} (Waldmeier, 1961), is one of the most important prediction items because the solar cycle modulates a number of Earth environment parameters. The prediction could be done with medium term methods, namely a few months in advance; or with long term methods, mostly based on precursors (see for example Lantos, 2006).

The methods of prediction based on precursors are generally obtained by a linear regression analysis between the maximum amplitude of the solar cycle RI_{max}, and an index of the precursor. Statistics is applied to the past cycles for which the precursor is available. Three kinds of precursors have been proposed. Some are based on solar measurements (Bravo and Otaola, 1989; Wilson, 1994), and others are based on geomagnetic indices (Lantos and Richard, 1998) or on characteristics of the RI_{12} time profile. A well-known example of the latter class is the minimum of RI_{12} between two cycles, which is a precursor of the next maximum RI_{max} (Sargent, 1978). The purpose of the present work is to revisit one of the best precursor methods based on RI_{12} time profile, in order to improve its efficiency and to discuss its advantages especially when combined with other precursors and with medium-term prediction methods.

2. Predicting Maximum of a Cycle from Calculated Skewness: Method (*a*)

Ramaswany (1977) has shown that the skewness of a cycle (a classical asymmetry coefficient in statistics) can be used as a precursor of the maximum of the next cycle. We consider the skewness γ:

$$\gamma = \mu_3/\sigma^3 = \mu_3/\mu_2^{3/2}$$

where μ_3 is the third moment about the mean, σ is the standard deviation and μ_2 is the second moment about the mean (variance). Table I gives the skewness of cycles 1 (1755-1766) to 22 (1986-1996). Positive skewness means faster rise and slower decline of the time profile. The parameter is calculated with the time profiles of RI_{12} when available. Indeed the first few cycles were not observed as completely as the recent ones (Waldmeier, 1961). Nevertheless the skewness is a global characteristic of cycles, not sensitive to precise determination of RI_{12}. According to Ramaswany's suggestion, the linear regression is applied to the ratio **R** of the maximum of the following cycle to the maximum of the given cycle, the skewness being the independent variable. When applied to all cycles 1 to 22, the correlation coefficient amounts to 0.753. This is already a rather reliable method which can be easily improved.

The modification we introduce here is a separation between even- and odd-numbered cycles because this improves appreciably the fits. Figure 1a shows the regression line for even-numbered cycles and Figure 1b shows the same for odd-numbered cycles. The figures show that the distributions are quite different.

Table I: Skewness and duration of the ascending phase (in months) for cycles 1 to 23.

Cycle	Skewn.	Dur. asc.	Cycle	Skewn.	Dur. asc.	Cycle	Skewn.	Dur. asc.
1	-0.077	75	9	0.235	55	17	0.299	43
2	0.195	40	10	0.346	50	18	0.273	39
3	0.531	35	11	0.646	41	19	0.581	47
4	0.632	41	12	0.414	60	20	0.330	49
5	0.047	82	13	0.640	47	21	0.299	44
6	0.078	70	14	0.204	49	22	0.419	34
7	-0.219	79	15	0.314	49	23		47
8	0.456	40	16	0.262	57			

In the regression analysis, the skewness of cycle 22 is not included in order to use this precursor to predict the present cycle 23. When the observed cycle is even-numbered, the correlation coefficient is –0.857: when the observed cycle is odd-numbered, the correlation coefficient is –0.831. When the errors on the predicted RI_{max} are considered for odd- and even-numbered cycles together, the standard error of estimate is found to be 22.1. The linear regression formulae are:

$$\mathbf{R} = -\,2.1092\,\gamma + 1.9418 \text{ when } \gamma \text{ corresponds to an even-numbered cycle} \qquad (1)$$

$$\mathbf{R} = -\,1.2552\,\gamma + 1.3570 \text{ when } \gamma \text{ corresponds to an odd-numbered cycle .} \qquad (2)$$

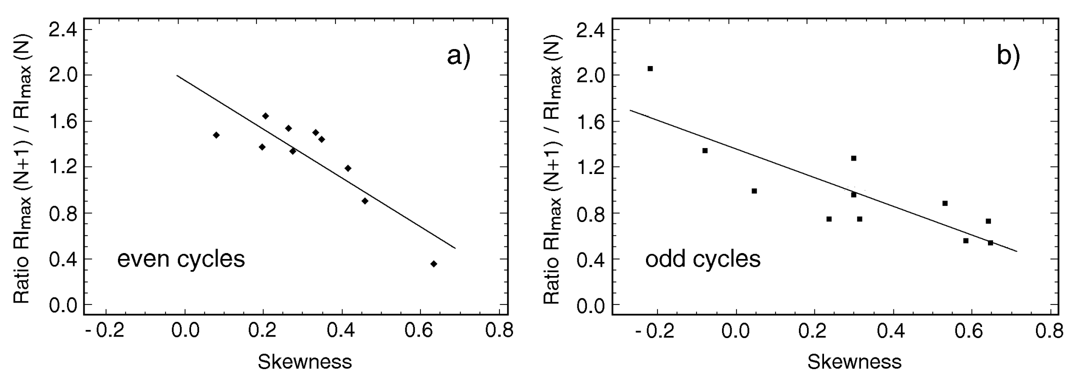

Figure 1: Linear regression of the ratio of the maximum of the next cycle to the observed one as a function of the skewness of the observed cycle: (a) for even-numbered cycles and (b) for odd-numbered cycles.

The relation (1), with a skewness for cycle 22 equal to 0.419 predicts for cycle 23 a maximum of 168 ± 22, higher than the observed value of 120.7, but rather similar to the predictions from most of the other precursors. As discussed in Lantos (2006), the poor quality of the predictions of those precursors is specific to cycle 23: other cycles are more reliably predicted. Indeed Figure 2 shows a diagram comparing the maximum RI_{max} predicted with the method described above and the maximum RI_{max} observed. The line is the bisecting line. The point corresponding to cycle 23 is indicated.

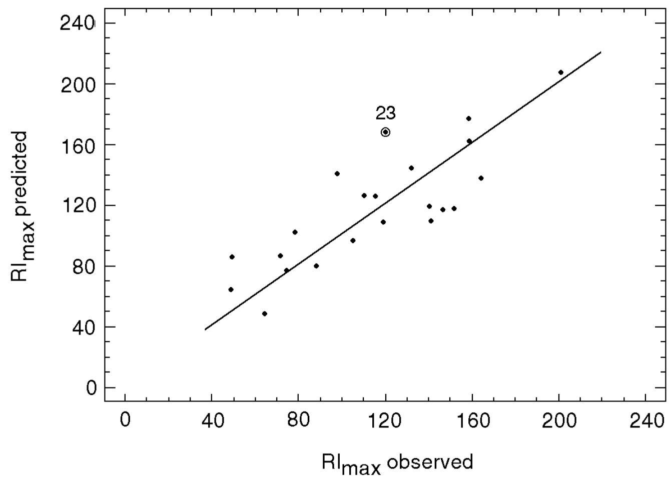

Figure 2: Comparison of the predicted maximum (vertical axis) and of the observed maximum (horizontal axis) for cycles 1 to 23.The bisecting line is plotted and the location of cycle 23 is indicated.

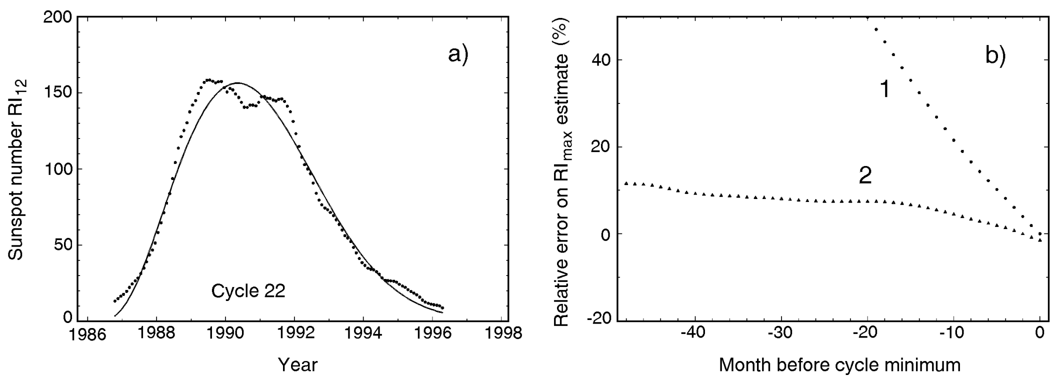

Figure 3a: Time profile of cycle number 22 in RI_{12} (dots) in comparison with the HWR function for cycle 22. Figure 3b: Relative errors on the predicted RI_{max} when the cycle is not finished. The horizontal axis is the time (in months) before the end of the cycle. Curve 1 shows the error when the missing data are not replenished. Curve 2 shows the error when the missing data are replaced with the HWR function.

The skewness is available at the end of the sunspot cycle. The epoch of the minimum is known at least 6 months after its actual occurrence because of the use of smoothed indices. Nevertheless the calculation of the skewness could be made slightly in advance. If the cycle is not finished, the missing values would modify appreciably the calculated skewness and thus the predicted RI_{max}. The result is much better if the observed points are completed with points deduced with a method like that proposed by Hathaway, Wilson, and Reichmann (1994) to fit the sunspot cycle profiles (hereafter called the HWR function). Figure 3a compares the observed time profile (dots) of cycle 22 with the calculated profile (line). Figure 3b gives, as a function of the number of months missing (or completed) at the end of the cycle 22, the relative error on the estimation of RI_{max} of cycle 23 derived from the skewness of previous cycle. Curve 1 gives the relative error when the solar cycle is shortened without extrapolation. Curve 2 shows the case where the calculated profile of Figure 3a is used to complete the RI_{12} cycle profile. A reasonable range of extrapolation is about one year. Indeed when cycles 1 to 22 are considered, the data completed in this way reduce the error to less than 10 % for 15 out of 22 cycles and to less than 16 % for all the cycles.

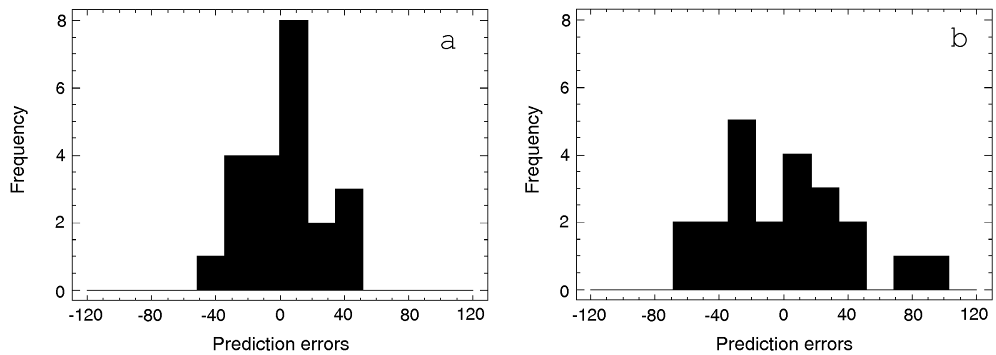

Figure 4: (a) Histogram of the prediction errors when the skewness is calculated at the end of the previous cycle. (b) Histogram of the prediction errors when the skewness is predicted using the observed RI_{max} of the cycle in progress.

3. Method (*a* o *b*): Composition of Calculated and Predicted Skewness

The skewness of a cycle could be predicted when the maximum of the cycle is observed. Indeed the duration of the ascending phase is correlated to the asymmetry and thus to the skewness of the cycle. Table I gives the duration of the ascending phase for cycles 1 to 23. We apply a linear regression with the skewness γ as dependent variable to cycles 1 to 22 (method (*b*)) and the relation we obtain is:

$$\gamma = -0.01226 \times A + 0.9413$$

where A is the duration of the ascending phase expressed in months. The correlation coefficient is –0.756 and the standard error of estimate is 0.152. The prediction of the skewness combined with the prediction of the next maximum (Equations (1) and (2)) leads to 132.2 as the prediction of RI_{max} for cycle 23 (the observed value was 120.7) and the prediction of RI_{max} for future cycle 24 is 108.4 (cycle 24 is foreseen to start at the beginning of 2007).

The composition of methods gives logically larger errors of prediction of RI_{max} than when the skewness is calculated from the RI_{12} time profiles. Figures 4a and 4b show the histograms of errors when the maximum of the next cycle, RI_{max}, is predicted. Figure 4a corresponds to the skewness calculated at the end of the previous cycle (method (*a*)). Figure 4b corresponds to the skewness predicted from the duration of the ascending phase of the current cycle: method (*a* o *b*) is the composition of the two methods, beginning with method (*b*). Comparison of figures shows that the distribution is indeed somewhat larger in the second case; two cycles (22 and 23) particularly showing a large error of prediction. With method (*a*) the standard error of estimate on the maximum sunspot index is 23.71 and with method (*a* o *b*) the standard error is 38.9.

4. Discussion

The two methods using the skewness as a precursor of the maximum of the next cycle could be compared to the other precursor methods based on the RI_{12} time profile. A method based on the slope at the inflexion point of the ascending phase has been proposed by Lantos (2000). For solar cycles 9 (1843-1855) to 22 the correlation coefficient is found to be 0.88 and the standard error of estimate is 19.7. Thus the correlation is high and the method is reliable. The correlation coefficient is similar to those of method (*a*) (r = –0.857 and r = –0.831), but the precursor is available quite late, because the inflexion point occurs in the middle of the ascending phase of the cycle to be predicted. For cycle 23 the method predicted 103±20 r.m.s. for RI_{max} (while the observed value was 120.7).

As mentioned in the introduction, the minimum of the sunspot index, RI_{min}, is frequently used as a precursor, in particular when bivariate analysis is performed (Sargent, 1978; Kane, 1989; Wilson; 1998; Lantos and Richard, 1998). When the cycles with reliable RI_{12} measurements are taken into account (cycles 8 to 22), the correlation coefficient is found to be 0.47 and the standard error of estimate is 35.4. For cycle 23 the method predicted 133 ± 35 r.m.s. for RI_{max}. With method (*a* o *b*) the standard error is 38.9 and thus method (*a* o *b*) shows performance similar to that of the method using RI_{min} as a precursor.

It has been shown recently (Lantos, 2006) that the combination of precursor methods not only improve the statistics, but also reduces the danger of false predictions (with errors larger than 20 %) encountered (rarely) in all the methods. The combination is efficient if the adopted precursors are statistically independent. Thus it is important to find precursors like skewness which are physically independent from other precursors that are mostly related to geomagnetic activity. Lantos (2006) has also shown that a frequently used medium term prediction method, the McNish and Lincoln method (McNish and Lincoln, 1949), could be greatly improved by using it in combination with a group of precursor methods including the method (*a*) described here.

5. Conclusion

We have revisited a cycle maximum precursor, the skewness γ, proposed by Ramaswamy in 1977. The skewness of a cycle *N* is a good precursor of the maximum of the solar cycle *N*+1. The original method has been improved by separating odd- and even-numbered cycles. The time at which the precursor is available could be easily anticipated by about one year and a prediction of the skewness could even provide prediction of RI_{max} as much as 11 years in advance. If the skewness is calculated with the observed time profiles of RI_{12} (method (*a*)), the reliability is similar to that of some of the best precursor methods like the method based on the inflexion point of the ascending phase of the cycle. If the skewness itself is predicted (method (*a* o *b*)), the prediction of the maximum of cycle *N*+1 is available as soon as the maximum of cycle *N* is observed. This makes this prediction the earliest of the methods proposed up to now, about 11 years in advance to the maximum of cycle *N*+1. The statistical reliability of the prediction is lower than that of method (*a*). With method (*a* o *b*), the maximum of the coming cycle 24 is predicted to be 108 ± 38 (the expected maximum will be in 2011). As a conclusion of this work, the precursor based on the skewness of the cycle profiles, after about thirthy years of absence in the literature, appears as a useful precursor of the maximum amplitude of the solar cycle. It deserves a place in a small group of reliable solar cycle precursors (see Lantos, 2006).

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